

CHAPTER 2 LOAD FLOW ANALYSIS

THE POWER FLOW PROBLEM

Input Data:

1. Bus Data
2. Transmission Line Data
Include per-unit equivalent π circuit series impedance Z' and shunt admittance Y' , the two buses to which the line is connected, and maximum MVA rating.
3. Transformer Data
Include per-unit winding impedances Z , the per-unit exciting branch admittance Y , the buses to which the windings are connected, and maximum MVA ratings. For tap-changing transformers, maximum tap settings.

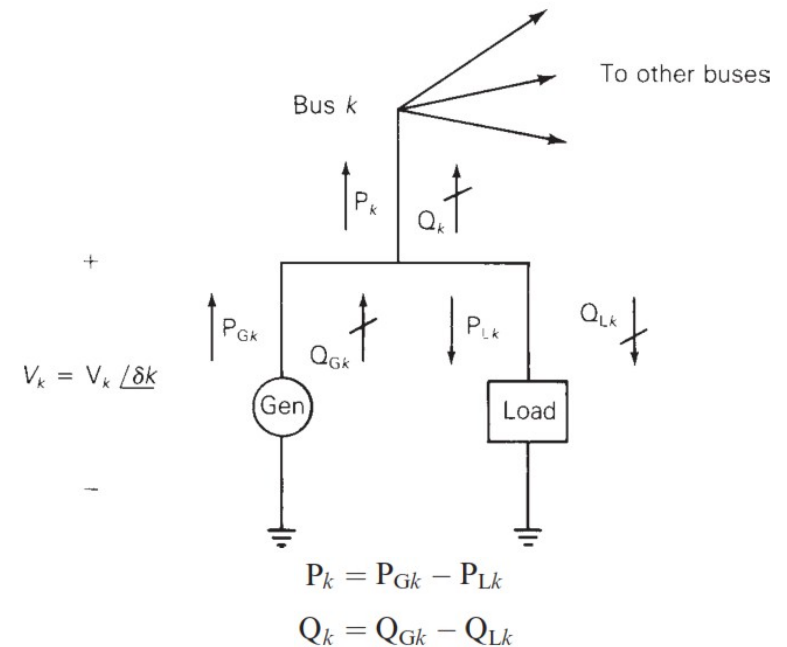
Bus Variables (at each bus k):

- V_k - voltage magnitude
- δ_k - phase angle
- P_k - net real power
- Q_k - net reactive power

Bus types:

1. **Swing bus (or slack bus)**—There is only one swing bus, which for convenience is numbered bus 1. The swing bus is a reference bus for which $V_1 \angle \delta_1$, typically $1 \angle 0^\circ$ per unit, is input data. The power-flow program computes P_1 and Q_1 .
2. **Load (PQ) bus**— P_k and Q_k are input data. The power-flow program computes V_k and δ_k . Most buses in a typical power-flow program are load buses.
3. **Voltage controlled (PV) bus**— P_k and V_k are input data. The power-flow program computes Q_k and δ_k . Examples are buses to which generators, switched shunt capacitors, or static var systems are connected. Maximum and minimum var limits Q_{Gkmax} and Q_{Gkmin} that this equipment can supply are also input data. If an upper or lower reactive power limit is reached, then

the reactive power output of the generator is held at the limit, and the bus is modeled as a PQ bus. Another example is a bus to which a tap-changing transformer is connected; the power-flow program then computes the tap setting.



Bus admittance matrix, Y_{bus} :

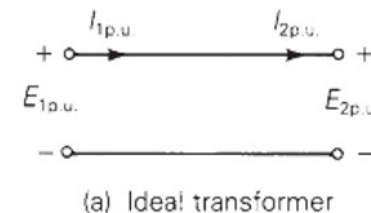
Constructed from the line and transformer input data.

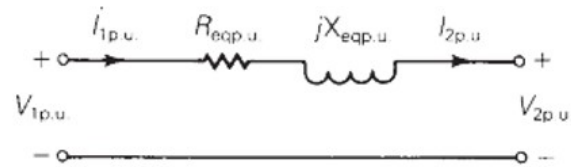
Diagonal elements: Y_{kk} = sum of admittances connected to bus k

Off-diagonal elements: Y_{kn} = -(sum of admittances connected between buses k and n)

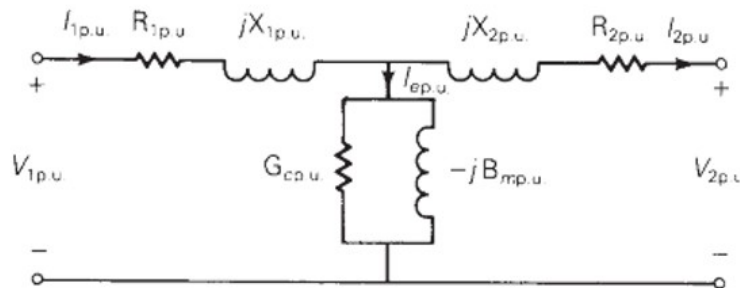
Network Element Representation

Two-winding Transformers:

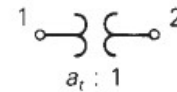




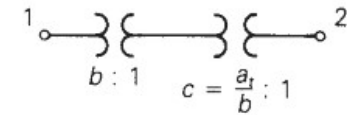
(b) Neglecting exciting current



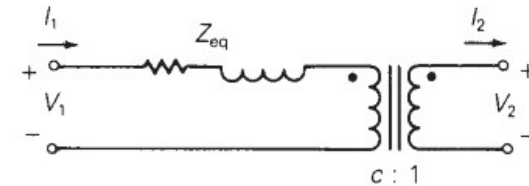
(c) Complete representation



(a) Single-line diagram



(b) Represented as two transformers in series



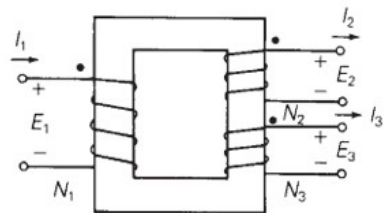
(c) Per-unit equivalent circuit
(Per-unit impedance is shown)

EXAMPLE 6.9

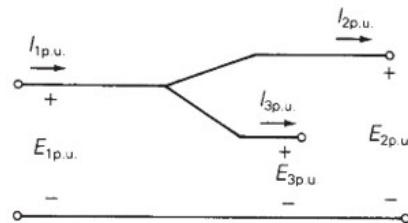
Power-flow input data and Y_{bus}

Figure 6.2 shows a single-line diagram of a five-bus power system. Input data are given in Tables 6.1, 6.2, and 6.3. As shown in Table 6.1, bus 1, to which a generator is connected, is the swing bus. Bus 3, to which a generator and a load are connected, is a voltage-controlled bus. Buses 2, 4, and 5 are load buses. Note that the loads at buses 2 and 3 are inductive since $Q_2 = -Q_{L2} = -2.8$ and $-Q_{L3} = -0.4$ are negative.

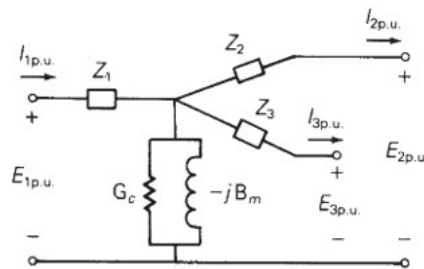
For each bus k , determine which of the variables V_k , δ_k , P_k , and Q_k are input data and which are unknowns. Also, compute the elements of the second row of Y_{bus} .



(a) Basic core and coil configuration



(b) Per-unit equivalent circuit—ideal transformer



(c) Per-unit equivalent circuit—practical transformer

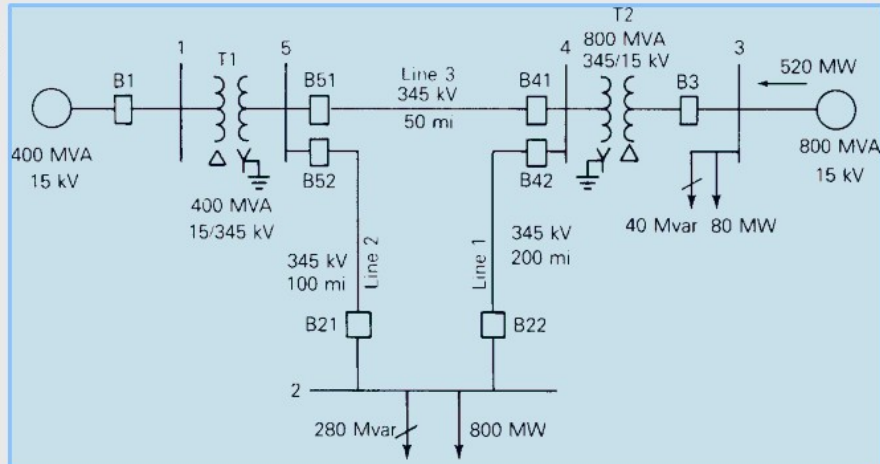
Three- Winding Transformer:

Tap-Changing Transformer:

FIGURE 6.2

Single-line diagram for
Example 6.9

SOLUTION The input data and unknowns are listed in Table 6.4. For bus 1, the swing bus, P_1 and Q_1 are unknowns. For bus 3, a voltage-controlled bus,

**TABLE 6.1**

Bus input data for
Example 6.9*

Bus	Type	V per unit	δ degrees	P_G per unit	Q_G per unit	P_L per unit	Q_L per unit	Q_{Gmax} per unit	Q_{Gmin} per unit
1	Swing	1.0	0	—	—	0	0	—	—
2	Load	—	—	0	0	8.0	2.8	—	—
3	Constant voltage	1.05	—	5.2	—	0.8	0.4	4.0	-2.8
4	Load	—	—	0	0	0	0	—	—
5	Load	—	—	0	0	0	0	—	—

* $S_{base} = 100$ MVA, $V_{base} = 15$ kV at buses 1, 3, and 345 kV at buses 2, 4, 5

TABLE 6.2

Line input data for
Example 6.9

Bus-to-Bus	R' per unit	X' per unit	G' per unit	B' per unit	Maximum MVA per unit
2-4	0.0090	0.100	0	1.72	12.0
2-5	0.0045	0.050	0	0.88	12.0
4-5	0.00225	0.025	0	0.44	12.0

TABLE 6.3

Transformer input data
for Example 6.9

Bus-to-Bus	R per unit	X per unit	G_c per unit	B_m per unit	Maximum MVA per unit	Maximum TAP Setting per unit
1-5	0.00150	0.02	0	0	6.0	—
3-4	0.00075	0.01	0	0	10.0	—

TABLE 6.4

Input data and
unknowns for
Example 6.9

Bus	Input Data	Unknowns
1	$V_1 = 1.0, \delta_1 = 0$	P_1, Q_1
2	$P_2 = P_{G2} - P_{L2} = -8$ $Q_2 = Q_{G2} - Q_{L2} = -2.8$	V_2, δ_2
3	$V_3 = 1.05$ $P_3 = P_{G3} - P_{L3} = 4.4$	Q_3, δ_3
4	$P_4 = 0, Q_4 = 0$	V_4, δ_4
5	$P_5 = 0, Q_5 = 0$	V_5, δ_5

Q_3 and δ_3 are unknowns. For buses 2, 4, and 5, load buses, V_2, V_4, V_5 and $\delta_2, \delta_4, \delta_5$ are unknowns.

$$Y_{21} = Y_{23} = 0$$

$$Y_{24} = \frac{-1}{R'_{24} + jX'_{24}} = \frac{-1}{0.009 + j0.1} = -0.89276 + j9.91964 \text{ per unit}$$

$$= 9.95972/95.143^\circ \text{ per unit}$$

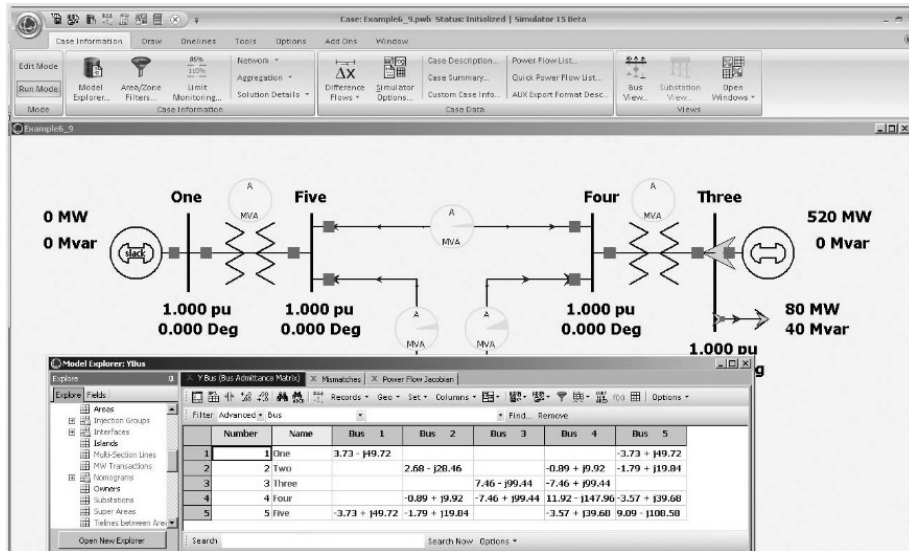
$$Y_{25} = \frac{-1}{R'_{25} + jX'_{25}} = \frac{-1}{0.0045 + j0.05} = -1.78552 + j19.83932 \text{ per unit}$$

$$= 19.9195/95.143^\circ \text{ per unit}$$

$$Y_{22} = \frac{1}{R'_{24} + jX'_{24}} + \frac{1}{R'_{25} + jX'_{25}} + j\frac{B'_{24}}{2} + j\frac{B'_{25}}{2}$$

$$= (0.89276 - j9.91964) + (1.78552 - j19.83932) + j\frac{1.72}{2} + j\frac{0.88}{2}$$

$$= 2.67828 - j28.4590 = 28.5847/-84.624^\circ \text{ per unit}$$



This five-bus power system is modeled in PowerWorld Simulator case Example 6.9 (see Figure 6.3). To view the input data, first click on the **Edit Mode** button (on the far left-hand side of the ribbon) to switch into the Edit mode (the Edit mode is used for modifying system parameters). Then by selecting the **Case Information** tab you can view tabular displays showing the various parameters for the system. For example, use **Network, Buses** to view the parameters for each bus, and **Network, Lines and Transformers** to view the parameters for the transmission lines and transformers. Fields shown in blue can be directly changed simply by typing over them, and those shown in green can be toggled by clicking on them. Note that the values shown on these displays match the values from Tables 6.1 to 6.3, except the power values are shown in actual MW/Mvar units.

The elements of Y_{bus} can also be displayed by selecting **Solution Details, Y_{bus}** . Since the Y_{bus} entries are derived from other system parameters, they cannot be changed directly. Notice that several of the entries are blank, indicating that there is no line directly connecting these two buses (a blank entry is equivalent to zero). For larger networks most of the elements of the Y_{bus} are zero since any single bus usually only has a few incident lines (such sparse matrices are considered in Section 6.8). The elements of the Y_{bus} can be saved in a Matlab compatible format by first right-clicking within the Y_{bus} matrix to display the local menu, and then selecting **Save Y_{bus} in Matlab Format** from the local menu.

Finally, notice that no flows are shown on the one-line because the nonlinear power-flow equations have not yet been solved. We cover the solution of these equations next. ■

Power-flow Equations

Using Y_{bus} , we can write nodal equations for a power system network, as follows:

$$I = Y_{bus} V \quad (6.4.3)$$

where I is the N vector of source currents injected into each bus and V is the N vector of bus voltages.

$$I_k = \sum_{n=1}^N Y_{kn} V_n \quad (6.4.4)$$

$$S_k = P_k + jQ_k = V_k I_k^* \quad (6.4.5)$$

$$P_k + jQ_k = V_k \left[\sum_{n=1}^N Y_{kn} V_n \right]^* \quad k = 1, 2, \dots, N \quad (6.4.6)$$

$$V_n = V_n e^{j\delta_n} \quad (6.4.7)$$

$$Y_{kn} = Y_{kn} e^{j\theta_{kn}} = G_{kn} + jB_{kn} \quad k, n = 1, 2, \dots, N \quad (6.4.8)$$

$$P_k + jQ_k = V_k \sum_{n=1}^N Y_{kn} V_n e^{j(\delta_k - \delta_n - \theta_{kn})} \quad (6.4.9)$$

$$P_k = V_k \sum_{n=1}^N Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn}) \quad (6.4.10)$$

$$Q_k = V_k \sum_{n=1}^N Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn}) \quad k = 1, 2, \dots, N \quad (6.4.11)$$

$$P_K = V_K \sum_{n=1}^N V_n [G_{kn} \cos(\delta_k - \delta_n) + B_{kn} \sin(\delta_k - \delta_n)] \quad (6.4.12)$$

$$Q_K = V_K \sum_{n=1}^N V_n [G_{kn} \sin(\delta_k - \delta_n) - B_{kn} \cos(\delta_k - \delta_n)] \quad k = 1, 2, \dots, N \quad (6.4.13)$$

POWER-FLOW SOLUTION BY GAUSS-SEIDEL

For each load bus, I_k can be calculated from (6.4.5), giving

$$I_k = \frac{P_k - jQ_k}{V_k^*} \quad (6.5.1)$$

Applying the Gauss-Seidel method,

$$V_k(i+1) = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k^*(i)} - \sum_{n=1}^{k-1} Y_{kn} V_n(i+1) - \sum_{n=k+1}^N Y_{kn} V_n(i) \right] \quad (6.5.2)$$

Equation (6.5.2) can be applied twice during each iteration for load buses, first using $V_k^*(i)$, then replacing $V_k^*(i)$, by $V_k^*(i+1)$ on the right side of (6.5.2).

For a voltage-controlled bus, Q_k is unknown, but can be calculated from (6.4.11), giving

$$Q_k = V_k(i) \sum_{n=1}^N Y_{kn} V_n(i) \sin[\delta_k(i) - \delta_n(i) - \theta_{kn}] \quad (6.5.3)$$

Also,

$$Q_{Gk} = Q_k + Q_{Lk}$$

If the calculated value of Q_{Gk} does not exceed its limits, then Q_k is used in (6.5.2) to calculate $V_k(i+1) = V_k(i+1)/\delta_k(i+1)$. Then the magnitude $V_k(i+1)$ is changed to V_k , which is input data for the voltage-controlled bus. Thus we use (6.5.2) to compute only the angle $\delta_k(i+1)$ for voltage-controlled buses.

If the calculated value exceeds its limit $Q_{Gk\max}$ or $Q_{Gk\min}$ during any iteration, then the bus type is changed from a voltage-controlled bus to a load bus, with Q_{Gk} set to its limit value. Under this condition, the voltage-controlling device (capacitor bank, static var system, and so on) is not capable of maintaining V_k as specified by the input data. The power-flow program then calculates a new value of V_k .

For the swing bus, denoted bus 1, V_1 and δ_1 are input data. As such, no iterations are required for bus 1. After the iteration process has converged, one pass through (6.4.10) and (6.4.11) can be made to compute P_1 and Q_1 .

EXAMPLE 6.10 Power-flow solution by Gauss–Seidel

For the power system of Example 6.9, use Gauss–Seidel to calculate $V_2(1)$, the phasor voltage at bus 2 after the first iteration. Use zero initial phase angles and 1.0 per-unit initial voltage magnitudes (except at bus 3, where $V_3 = 1.05$) to start the iteration procedure.

SOLUTION Bus 2 is a load bus. Using the input data and bus admittance values from Example 6.9 in (6.5.2),

$$\begin{aligned} V_2(1) &= \frac{1}{Y_{22}} \left\{ \frac{P_2 - jQ_2}{V_2^*(0)} - [Y_{21} V_1(1) + Y_{23} V_3(0) + Y_{24} V_4(0) + Y_{25} V_5(0)] \right\} \\ &= \frac{1}{28.5847/-84.624^\circ} \left\{ \frac{-8 - j(-2.8)}{1.0/0^\circ} \right. \\ &\quad \left. - [(-1.78552 + j19.83932)(1.0) + (-0.89276 + j9.91964)(1.0)] \right\} \\ &= \frac{(-8 + j2.8) - (-2.67828 + j29.7589)}{28.5847/-84.624^\circ} \\ &= 0.96132/-16.543^\circ \text{ per unit} \end{aligned}$$

Next, the above value is used in (6.5.2) to recalculate $V_2(1)$:

$$\begin{aligned} V_2(1) &= \frac{1}{28.5847/-84.624^\circ} \left\{ \frac{-8 + j2.8}{0.96132/16.543^\circ} \right. \\ &\quad \left. - [-2.67828 + j29.75829] \right\} \\ &= \frac{-4.4698 - j24.5973}{28.5847/-84.624^\circ} = 0.87460/-15.675^\circ \text{ per unit} \end{aligned}$$

Computations are next performed at buses 3, 4, and 5 to complete the first Gauss–Seidel iteration.

To see the complete convergence of this case, open PowerWorld Simulator case Example 6_10. By default, PowerWorld Simulator uses the Newton–Raphson method described in the next section. However, the case can be solved with the Gauss–Seidel approach by selecting **Tools, Solve, Gauss–Seidel Power Flow**. To avoid getting stuck in an infinite loop if a case does not converge, PowerWorld Simulator places a limit on the maximum number of iterations. Usually for a Gauss–Seidel procedure this number is quite high, perhaps equal to 100 iterations. However, in this example to demonstrate the convergence characteristics of the Gauss–Seidel method it has been set to a single iteration, allowing the voltages to be viewed after each iteration. To step through the solution one iteration at a time, just repeatedly select **Tools, Solve, Gauss–Seidel Power Flow**.

A common stopping criteria for the Gauss–Seidel is to use the scaled difference in the voltage from one iteration to the next (6.2.2). When this difference is below a specified convergence tolerance ε for each bus, the problem is considered solved. An alternative approach, implemented in PowerWorld Simulator, is to examine the real and reactive mismatch equations, defined as the difference between the right- and left-hand sides of (6.4.10) and (6.4.11). PowerWorld Simulator continues iterating until all the bus mismatches are below an MVA (or kVA) tolerance. When single-stepping through the solution, the bus mismatches can be viewed after each iteration on the **Case**

Information, Mismatches display. The solution mismatch tolerance can be changed on the Power Flow Solution page of the PowerWorld Simulator Options dialog (select **Tools, Simulator Options**, then select the **Power Flow Solution** category to view this dialog); the maximum number of iterations can also be changed from this page. A typical convergence tolerance is about 0.5 MVA. ■

POWER-FLOW SOLUTION BY NEWTON–RAPHSON

Equations (6.4.10) and (6.4.11) are nonlinear equations and of the form $\mathbf{y} = \mathbf{f}(\mathbf{x})$ whose solution by Newton–Raphson at the i^{th} iteration is given by

$$\mathbf{x}(i+1) = \mathbf{x}(i) + \mathbf{J}^{-1}(i)\{\mathbf{y} - \mathbf{f}[\mathbf{x}(i)]\} \quad (6.3.9)$$

where

$$\mathbf{J}(i) = \left. \frac{d\mathbf{f}}{d\mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}(i)} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_N} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_N} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_N}{\partial x_1} & \frac{\partial f_N}{\partial x_2} & \cdots & \frac{\partial f_N}{\partial x_N} \end{bmatrix}_{\mathbf{x}=\mathbf{x}(i)} \quad (6.3.10)$$

For equations (6.4.10) and (6.4.11)

$$\mathbf{x} = \begin{bmatrix} \delta \\ \mathbf{V} \end{bmatrix} = \begin{bmatrix} \delta_2 \\ \vdots \\ \delta_N \\ V_2 \\ \vdots \\ V_N \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} = \begin{bmatrix} P_2 \\ \vdots \\ P_N \\ Q_2 \\ \vdots \\ Q_N \end{bmatrix} \quad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{P}(\mathbf{x}) \\ \mathbf{Q}(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} P_2(\mathbf{x}) \\ \vdots \\ P_N(\mathbf{x}) \\ Q_2(\mathbf{x}) \\ \vdots \\ Q_N(\mathbf{x}) \end{bmatrix} \quad (6.6.1)$$

V , P , and Q terms are all in per-unit and δ terms are in radians. The swing bus variables δ_1 and V_1 are omitted from (6.6.1), since they are already known. Equations (6.4.10) and (6.4.11) then have the following form:

$$y_k = P_k = P_k(\mathbf{x}) = V_k \sum_{n=1}^N Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn}) \quad (6.6.2)$$

$$y_{k+N} = Q_k = Q_k(\mathbf{x}) = V_k \sum_{n=1}^N Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

$$k = 2, 3, \dots, N \quad (6.6.3)$$

TABLE 6.5

Elements of the
Jacobian matrix

$n \neq k$

$$J1_{kn} = \frac{\partial P_k}{\partial \delta_n} = V_k Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

$$J2_{kn} = \frac{\partial P_k}{\partial V_n} = V_k Y_{kn} \cos(\delta_k - \delta_n - \theta_{kn})$$

$$J3_{kn} = \frac{\partial Q_k}{\partial \delta_n} = -V_k Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$J4_{kn} = \frac{\partial Q_k}{\partial V_n} = V_k Y_{kn} \sin(\delta_k - \delta_n - \theta_{kn})$$

$n = k$

$$J1_{kk} = \frac{\partial P_k}{\partial \delta_k} = -V_k \sum_{\substack{n=1 \\ n \neq k}}^N Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

$$J2_{kk} = \frac{\partial P_k}{\partial V_k} = V_k Y_{kk} \cos \theta_{kk} + \sum_{\substack{n=1 \\ n \neq k}}^N Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$J3_{kk} = \frac{\partial Q_k}{\partial \delta_k} = V_k \sum_{\substack{n=1 \\ n \neq k}}^N Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$$

$$J4_{kk} = \frac{\partial Q_k}{\partial V_k} = -V_k Y_{kk} \sin \theta_{kk} + \sum_{\substack{n=1 \\ n \neq k}}^N Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn})$$

$$k, n = 2, 3, \dots, N$$

The Jacobian matrix of (6.3.10) has the form

$$\mathbf{J} = \begin{array}{c|c} \begin{array}{cc} \mathbf{J1} & \mathbf{J2} \\ \hline \frac{\partial \mathbf{P}_2}{\partial \delta_2} & \dots & \frac{\partial \mathbf{P}_2}{\partial \delta_N} \\ \vdots & & \vdots \\ \frac{\partial \mathbf{P}_N}{\partial \delta_2} & \dots & \frac{\partial \mathbf{P}_N}{\partial \delta_N} \end{array} & \begin{array}{cc} \frac{\partial \mathbf{P}_2}{\partial \mathbf{V}_2} & \dots & \frac{\partial \mathbf{P}_2}{\partial \mathbf{V}_N} \\ \vdots & & \vdots \\ \frac{\partial \mathbf{P}_N}{\partial \mathbf{V}_2} & \dots & \frac{\partial \mathbf{P}_N}{\partial \mathbf{V}_N} \end{array} \\ \hline \begin{array}{cc} \mathbf{J3} & \mathbf{J4} \\ \hline \frac{\partial \mathbf{Q}_2}{\partial \delta_2} & \dots & \frac{\partial \mathbf{Q}_2}{\partial \delta_N} \\ \vdots & & \vdots \\ \frac{\partial \mathbf{Q}_N}{\partial \delta_2} & \dots & \frac{\partial \mathbf{Q}_N}{\partial \delta_N} \end{array} & \begin{array}{cc} \frac{\partial \mathbf{Q}_2}{\partial \mathbf{V}_2} & \dots & \frac{\partial \mathbf{Q}_2}{\partial \mathbf{V}_N} \\ \vdots & & \vdots \\ \frac{\partial \mathbf{Q}_N}{\partial \mathbf{V}_2} & \dots & \frac{\partial \mathbf{Q}_N}{\partial \mathbf{V}_N} \end{array} \end{array} \quad (6.6.4)$$

Equation (6.6.4) is partitioned into four blocks. The partial derivatives in each block, derived from (6.6.2) and (6.6.3), are given in Table 6.5.

We now apply to the power-flow problem the four Newton–Raphson steps outlined in Section 6.3, starting with $\mathbf{x}(i) = \begin{bmatrix} \delta(i) \\ \mathbf{V}(i) \end{bmatrix}$ at the i th iteration.

Newton-Raphson Steps:

STEP 1 Use (6.6.2) and (6.6.3) to compute

$$\Delta \mathbf{y}(i) = \begin{bmatrix} \Delta \mathbf{P}(i) \\ \Delta \mathbf{Q}(i) \end{bmatrix} = \begin{bmatrix} \mathbf{P} - \mathbf{P}[\mathbf{x}(i)] \\ \mathbf{Q} - \mathbf{Q}[\mathbf{x}(i)] \end{bmatrix} \quad (6.6.5)$$

STEP 2 Use the equations in Table 6.5 to calculate the Jacobian matrix.

STEP 3 Use Gauss elimination and back substitution to solve

$$\begin{bmatrix} \mathbf{J1}(i) & \mathbf{J2}(i) \\ \hline \mathbf{J3}(i) & \mathbf{J4}(i) \end{bmatrix} \begin{bmatrix} \Delta \delta(i) \\ \Delta \mathbf{V}(i) \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{P}(i) \\ \Delta \mathbf{Q}(i) \end{bmatrix} \quad (6.6.6)$$

STEP 4 Compute

$$\mathbf{x}(i+1) = \begin{bmatrix} \delta(i+1) \\ \mathbf{V}(i+1) \end{bmatrix} = \begin{bmatrix} \delta(i) \\ \mathbf{V}(i) \end{bmatrix} + \begin{bmatrix} \Delta \delta(i) \\ \Delta \mathbf{V}(i) \end{bmatrix} \quad (6.6.7)$$

Starting with initial value $\mathbf{x}(0)$, the procedure continues until convergence is obtained or until the number of iterations exceeds a specified maximum. Convergence criteria are often based on $\Delta \mathbf{y}(i)$ (called *power mismatches*) rather than on $\Delta \mathbf{x}(i)$ (phase angle and voltage magnitude mismatches).

For each voltage-controlled bus, the magnitude V_k is already known, and the function $Q_k(\mathbf{x})$ is not needed. Therefore, we could omit V_k from the $\mathbf{x}$ vector and Q_k from the $\mathbf{y}$ vector. We could also omit from the Jacobian matrix the column corresponding to partial derivatives with respect to V_k and the row corresponding to partial derivatives of $Q_k(\mathbf{x})$. Alternatively, rows and corresponding columns for voltage-controlled buses can be retained in the Jacobian matrix. Then during each iteration, the voltage magnitude $V_k(i+1)$ of each voltage-controlled bus is reset to V_k , which is input data for that bus.

At the end of each iteration, we compute $Q_k(\mathbf{x})$ from (6.6.3) and $Q_{Gk} = Q_k(\mathbf{x}) + Q_{Lk}$ for each voltage-controlled bus. If the computed value of Q_{Gk} exceeds its limits, then the bus type is changed to a load bus with Q_{Gk} set to its limit value. The power-flow program also computes a new value for V_k .

EXAMPLE 6.11 Jacobian matrix and power-flow solution by Newton–Raphson

Determine the dimension of the Jacobian matrix for the power system in Example 6.9. Also calculate $\Delta P_2(0)$ in Step 1 and $J_{124}(0)$ in Step 2 of the first Newton–Raphson iteration. Assume zero initial phase angles and 1.0 per-unit initial voltage magnitudes (except $V_3 = 1.05$).

SOLUTION Since there are $N = 5$ buses for Example 6.9, (6.6.2) and (6.6.3) constitute $2(N-1) = 8$ equations, for which $\mathbf{J}(i)$ has dimension 8×8 .

However, there is one voltage-controlled bus, bus 3. Therefore, V_3 and the equation for $Q_3(\mathbf{x})$ could be eliminated, with $\mathbf{J}(i)$ reduced to a 7×7 matrix.

From Step 1 and (6.6.2),

$$\begin{aligned}\Delta P_2(0) &= P_2 - P_2(\mathbf{x}) = P_2 - V_2(0)\{Y_{21}V_1 \cos[\delta_2(0) - \delta_1(0) - \theta_{21}] \\ &\quad + Y_{22}V_2 \cos[-\theta_{22}] + Y_{23}V_3 \cos[\delta_2(0) - \delta_3(0) - \theta_{23}] \\ &\quad + Y_{24}V_4 \cos[\delta_2(0) - \delta_4(0) - \theta_{24}] \\ &\quad + Y_{25}V_5 \cos[\delta_2(0) - \delta_5(0) - \theta_{25}]\}\end{aligned}$$

$$\begin{aligned}\Delta P_2(0) &= -8.0 - 1.0\{28.5847(1.0) \cos(84.624^\circ) \\ &\quad + 9.95972(1.0) \cos(-95.143^\circ) \\ &\quad + 19.9159(1.0) \cos(-95.143^\circ)\} \\ &= -8.0 - (-2.89 \times 10^{-4}) = -7.99972 \text{ per unit}\end{aligned}$$

From Step 2 and $\mathbf{J}_1$ given in Table 6.5

$$\begin{aligned}\mathbf{J}_{124}(0) &= V_2(0)Y_{24}V_4(0) \sin[\delta_2(0) - \delta_4(0) - \theta_{24}] \\ &= (1.0)(9.95972)(1.0) \sin[-95.143^\circ] \\ &= -9.91964 \text{ per unit}\end{aligned}$$

To see the complete convergence of this case, open PowerWorld Simulator case Example 6_11 (see Figure 6.4). Select **Case Information**, **Network**, **Mismatches** to see the initial mismatches, and **Case Information**, **Solution Details**, **Power Flow Jacobian** to view the initial Jacobian matrix. As is common in commercial power flows, PowerWorld Simulator actually includes rows in the Jacobian for voltage-controlled buses. When a generator is regulating its terminal voltage, this row corresponds to the equation setting the bus voltage magnitude equal to the generator voltage setpoint. However, if the generator hits a reactive power limit, the bus type is switched to a load bus.

To step through the New-Raphson solution, from the **Tools** Ribbon select **Solve**, **Single Solution—Full Newton**. Ordinarily this selection would perform a complete Newton-Raphson iteration, stopping only when all the mismatches are less than the desired tolerance. However, for this case, in order to allow you to see the solution process, the maximum number of iterations has been set to 1, allowing the voltages, mismatches and the Jacobian to be viewed after each iteration. To complete the solution, continue to select **Single Solution—Full Newton** until the solution convergence to the values shown in Tables 6.6, 6.7 and 6.8 (in about three iterations).

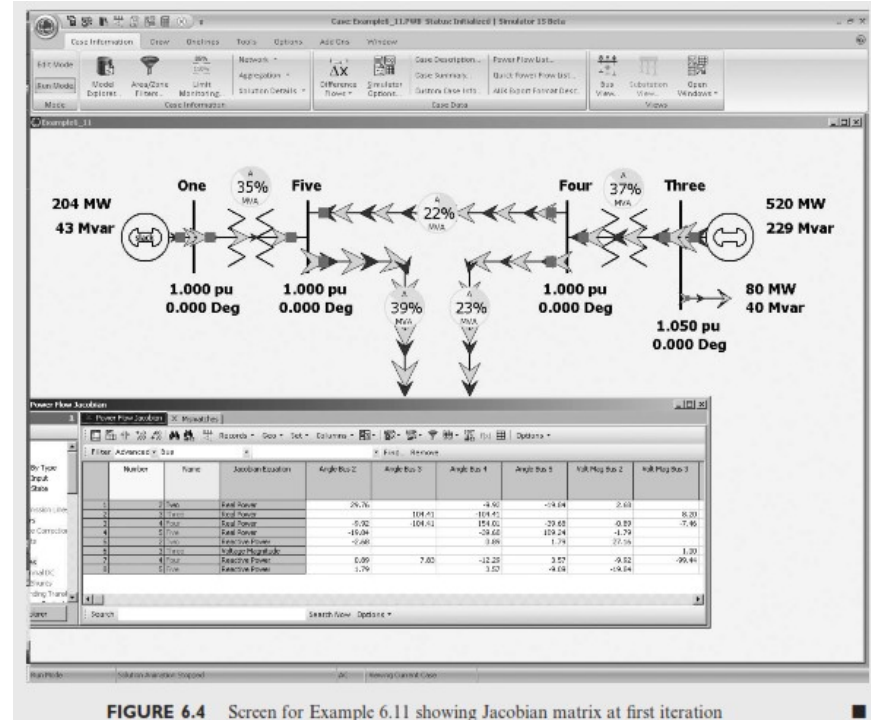


FIGURE 6.4 Screen for Example 6.11 showing Jacobian matrix at first iteration

TABLE 6.6

Bus output data for the power system given in Example 6.9

Bus #	Voltage Magnitude (per unit)	Phase Angle (degrees)	Generation		Load	
			PG (per unit)	QG (per unit)	PL (per unit)	QL (per unit)
1	1.000	0.000	3.948	1.144	0.000	0.000
2	0.834	-22.407	0.000	0.000	8.000	2.800
3	1.050	-0.597	5.200	3.376	0.800	0.400
4	1.019	-2.834	0.000	0.000	0.000	0.000
5	0.974	-4.548	0.000	0.000	0.000	0.000
TOTAL			9.148	4.516	8.800	3.200

TABLE 6.7

Line output data for the power system given in Example 6.9

Line #	Bus to Bus		P	Q	S
1	2	4	-2.920	-1.392	3.232
	4	2	3.036	1.216	3.272
2	2	5	-5.080	-1.408	5.272
	5	2	5.256	2.632	5.876
3	4	5	1.344	1.504	2.016
	5	4	-1.332	-1.824	2.260

TABLE 6.8

Transformer output data
for the power system
given in Example 6.9

Tran. #	Bus to Bus		P	Q	S
1	1	5	3.948	1.144	4.112
	5	1	-3.924	-0.804	4.004
2	3	4	4.400	2.976	5.312
	4	3	-4.380	-2.720	5.156

EXAMPLE 6.12 Power-flow program: change in generation

Using the power-flow system given in Example 6.9, determine the acceptable generation range at bus 3, keeping each line and transformer loaded at or below 100% of its MVA limit.

SOLUTION Load PowerWorld Simulator case Example 6.9. Select **Single Solution-Full Newton** to perform a single power-flow solution using the Newton–Raphson approach. Then view the **Case Information** displays to verify that the PowerWorld Simulator solution matches the solution shown in Tables 6.6, 6.7, and 6.8. Additionally, the pie charts on the one-lines show the percentage line and transformer loadings. Initially transformer T1, between buses 1 and 5, is loaded at about 68% of its maximum MVA limit, while transformer T2, between buses 3 and 4, is loaded at about 53%.

Next, the bus 3 generation needs to be varied. This can be done a number of different ways in PowerWorld Simulator. The easiest (for this example) is to use the bus 3 generator MW one-line field to manually change the generation (see Figure 6.5). Right-click on the “520 MW” field to the right of the bus 3 generator and select ‘**Generator Field Information**’ dialog to view the ‘**Generator Field Options**’ dialog. Set the “Delta Per Mouse Click” field to 10 and select OK. Small arrows are now visible next to this field on the one-line; clicking on the up arrow increases the generator’s MW output by 10 MW, while clicking on the down arrow decreases the generation by 10 MW. Select **Tools, Play** to begin the simulation. Increase the generation until the pie chart for the transformer from bus 3 to 4 is loaded to 100%. This occurs at about 1000 MW. Notice that as the bus 3 generation is increased the bus 1 slack generation decreases by a similar amount. Repeat the process, except now decreasing the generation. This unloads the transformer from bus 3 to 4, but increases the loading on the transformer from bus 1 to bus 5. The bus 1 to 5 transformer should reach 100% loading with the bus 3 generation equal to about 330 MW.

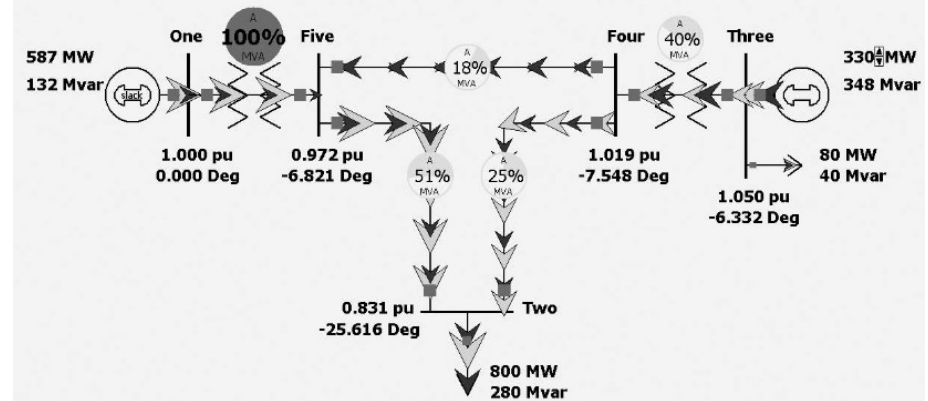


FIGURE 6.5 Screen for Example 6.12, Minimum Bus 3 Generator Loading

Voltage-controlled buses to which tap-changing or voltage-regulating transformers are connected can be handled by various methods. One method is to treat each of these buses as a load bus. The equivalent π circuit parameters (Figure 3.25) are first calculated with tap setting $c = 1.0$ for starting. During each iteration, the computed bus voltage magnitude is compared with the desired value specified by the input data. If the computed voltage is low (or high), c is increased (or decreased) to its next setting, and the parameters of the equivalent π circuit as well as Y_{bus} are recalculated. The procedure

continues until the computed bus voltage magnitude equals the desired value within a specified tolerance, or until the high or low tap-setting limit is reached. Phase-shifting transformers can be handled in a similar way by using a complex turns ratio $c = 1.0/\alpha$, and by varying the phase-shift angle α .

A method with faster convergence makes c a variable and includes it in the x vector of (6.6.1). An equation is then derived to enter into the Jacobian matrix [4].

The Newton-Raphson, or one of the derivative methods, are the preferred power-flow solution approaches.

EXAMPLE 6.13 Power-flow program: 37-bus system

To see a power-flow example of a larger system, open PowerWorld Simulator case Example 6_13 (see Figure 6.6). This case models a 37-bus, 9-generator power system containing three different voltage levels (345 kV, 138 kV, and 69 kV) with 57 transmission lines or transformers. The one-line can be panned by pressing the arrow keys, and it can be zoomed by pressing the <ctrl> with the up arrow key to zoom in or with the down arrow key to zoom out. Use **Tools, Play** to animate the one-line and **Tools, Pause** to stop the animation.

Determine the lowest per-unit voltage and the maximum line/transformer loading both for the initial case and for the case with the line from bus TIM69 to HANNAH69 out of service.

SOLUTION Use single solution to initially solve the power flow, and then **Case Information, Network, Buses...** to view a listing of all the buses in the case. To quickly determine the lowest per-unit voltage magnitude, left-click on the PU Volt column header to sort the column (clicking a second time reverses the sort). The lowest initial voltage magnitude is 0.9902 at bus DEMAR69. Next, select **Case Information, Network, Lines and Transformers...** to view the Line and Transformer Records display. Left-click on % of Max Limit to sort the lines by percentage loading. Initially the highest percentage loading is 64.9% on the line from UIUC69 to BLT69 circuit 1.

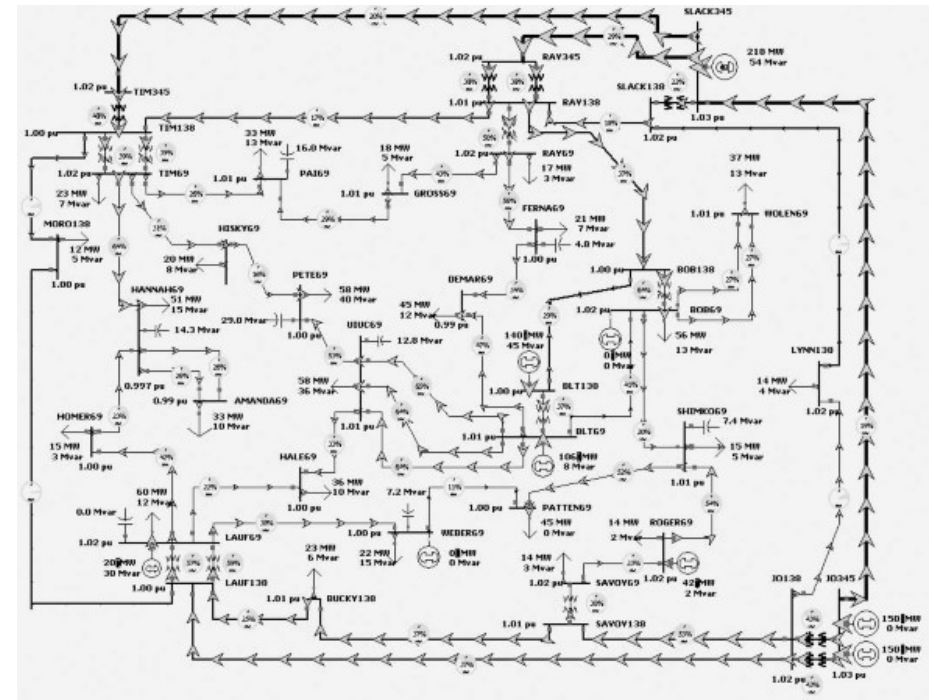


FIGURE 6.6 Screen for Example 6.13 showing the initial flows

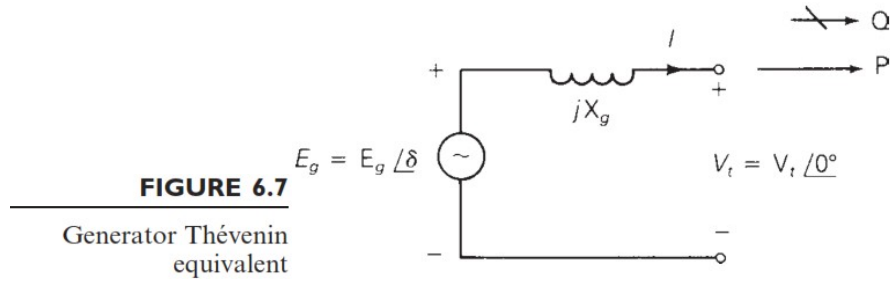
There are several ways to remove the TIM69 to HANNAH69 line. One approach is to locate the line on the Line and Transformer Records display and then double-click on the Status field to change its value. An alternative approach is to find the line on the one-line (it is in the upper-lefthand portion) and then click on one of its circuit breakers. Once the line is removed, use single solution to resolve the power flow. The lowest per-unit voltage is now 0.9104 at AMANDA69 and the highest percentage line loading is 134.8%, on the line from HOMER69 to LAUF69. Since there are now several bus and line violations, the power system is no longer at a secure operating point. Control actions and/or design improvements are needed to correct these problems.

CONTROL OF POWER FLOW

The following means are used to control system power flows:

1. Prime mover and excitation control of generators.
2. Switching of shunt capacitor banks, shunt reactors, and static var systems.

3. Control of tap-changing and regulating transformers.



A simple model of a generator operating under balanced steady-state conditions is the Thevenin equivalent shown in Figure 6.7. V_t is the generator terminal voltage, E_g is the excitation voltage, δ is the power angle, and X_g is the positive-sequence synchronous reactance. From the figure, the generator current is

$$I = \frac{E_g e^{j\delta} - V_t}{jX_g} \quad (6.7.1)$$

and the complex power delivered by the generator is

$$\begin{aligned} S = P + jQ &= V_t I^* = V_t \left(\frac{E_g e^{-j\delta} - V_t}{-jX_g} \right) \\ &= \frac{V_t E_g (j \cos \delta + \sin \delta) - jV_t^2}{X_g} \end{aligned} \quad (6.7.2)$$

The real and reactive powers delivered are then

$$P = \text{Re } S = \frac{V_t E_g}{X_g} \sin \delta \quad (6.7.3)$$

$$Q = \text{Im } S = \frac{V_t}{X_g} (E_g \cos \delta - V_t) \quad (6.7.4)$$

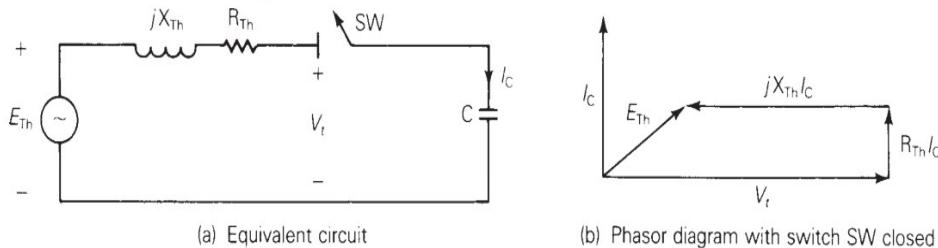


FIGURE 6.8 Effect of adding a shunt capacitor bank to a power system bus

Equation (6.7.3) shows that the real power P increases when the power angle δ increases. From an operational standpoint, when the prime mover increases the power input to the generator while the excitation voltage is held constant, the rotor speed increases. As the rotor speed increases, the power angle δ also increases, causing an increase in generator real power output P . There is also a decrease in reactive power output Q , given by (6.7.4). However, when δ is less than 15° , the increase in P is much larger than the decrease in Q . From the power-flow standpoint, an increase in prime-mover power corresponds to an increase in P at the constant-voltage bus to which the generator is connected. The power-flow program computes the increase in δ along with the small change in Q .

Equation (6.7.4) shows that reactive power output Q increases when the excitation voltage E_g increases. From the operational standpoint, when the generator exciter output increases while holding the prime-mover power constant, the rotor current increases. As the rotor current increases, the excitation voltage E_g also increases, causing an increase in generator reactive power output Q . There is also a small decrease in δ required to hold P constant in (6.7.3). From the power-flow standpoint, an increase in generator excitation corresponds to an increase in voltage magnitude at the constant-voltage bus to which the generator is connected. The power-flow program computes the increase in reactive power Q supplied by the generator along with the small change in δ .

Figure 6.8 shows the effect of adding a shunt capacitor bank to a power system bus. The system is modeled by its Thévenin equivalent. Before the capacitor bank is connected, the switch SW is open and the bus voltage equals E_{Th} . After the bank is connected, SW is closed, and the capacitor current I_c leads the bus voltage V_t by 90° . The phasor diagram shows that V_t is larger than E_{Th} when SW is closed. From the power-flow standpoint, the addition of a shunt capacitor bank to a load bus corresponds to the addition of a negative reactive load, since a capacitor absorbs negative reactive power. The power-flow program computes the increase in bus voltage magnitude along with the small change in δ . Similarly, the addition of a shunt reactor corresponds to the addition of a positive reactive load, wherein the power-flow program computes the decrease in voltage magnitude.

Tap-changing and voltage-magnitude-regulating transformers are used to control bus voltages as well as reactive power flows on lines to which they are connected. Similarly, phase-angle regulating transformers are used to control bus angles as well as real power flows on lines to which they are connected. Both tap-changing and regulating transformers are modeled by a transformer with an off-nominal turns ratio c (Figure 3.25). From the power-flow standpoint, a change in tap setting or voltage regulation corresponds to a change in c . The power-flow program computes the changes in Y_{bus} , bus voltage magnitudes and angles, and branch flows.

Besides the above controls, the power-flow program can be used to investigate the effect of switching in or out lines, transformers, loads, and generators. Proposed system changes to meet future load growth, including new transmission, new transformers, and new generation can also be investigated. Power-flow design studies are normally conducted by trial and error. Using engineering judgment, adjustments in generation levels and controls are made until the desired equipment loadings and voltage profile are obtained.

EXAMPLE 6.14 Power-flow program: effect of shunt capacitor banks

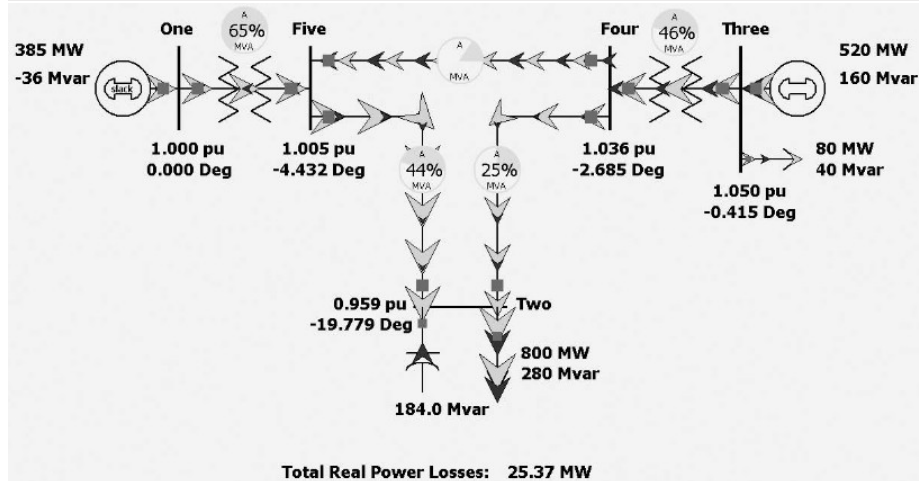


FIGURE 6.9 Screen for Example 6.14

Determine the effect of adding a 200-Mvar shunt capacitor bank at bus 2 on the power system in Example 6.9.

SOLUTION Open PowerWorld Simulator case Example 6_14 (see Figure 6.9). This case is identical to Example 6.9 except that a 200-Mvar shunt capacitor bank has been added at bus 2. Initially this capacitor is open. Click on the capacitor's circuit to close the capacitor and then solve the case. The capacitor increases the bus 2 voltage from 0.834 per unit to a more acceptable 0.959 per unit. The insertion of the capacitor has also substantially decreased the losses, from 34.84 to 25.37 MW.

Notice that the amount of reactive power actually supplied by the capacitor is only 184 Mvar. This discrepancy arises because a capacitor's reactive output varies with the square of the terminal voltage, $Q_{\text{cap}} = V_{\text{cap}}^2 / X_c$ (see 2.3.5). A capacitor's Mvar rating is based on an assumed voltage of 1.0 per unit. ■

EXAMPLE 6.15

PowerWorld Simulator Case Example 6_15 (see Figure 6.10), which modifies the Example 6.13 case by (1) opening one of the 138/69 kV transformers at the LAUF substation, and (2) opening the 69 kV transmission line between

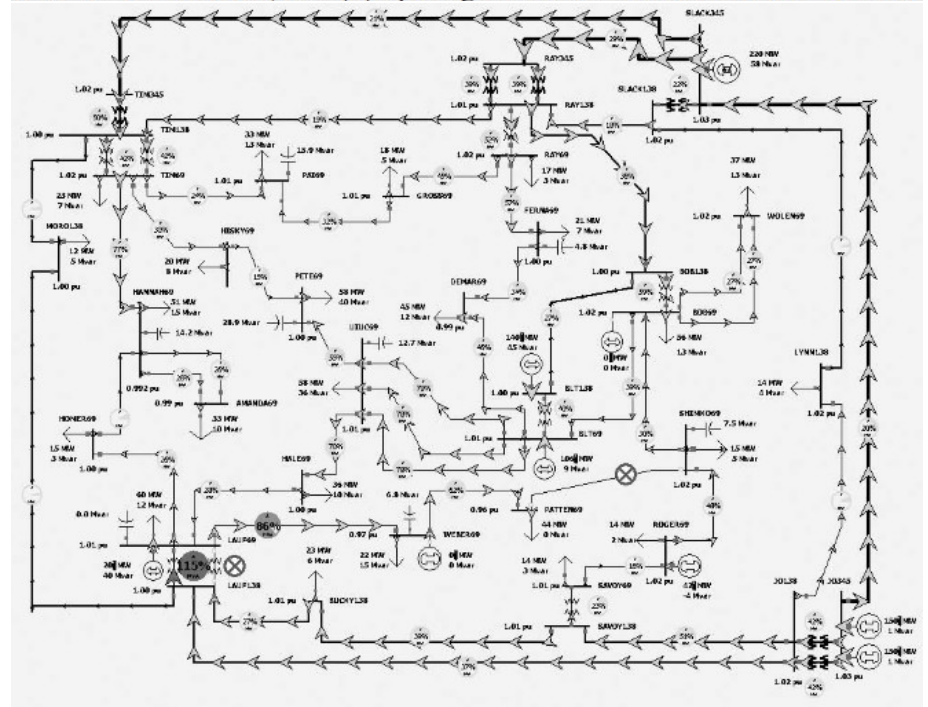
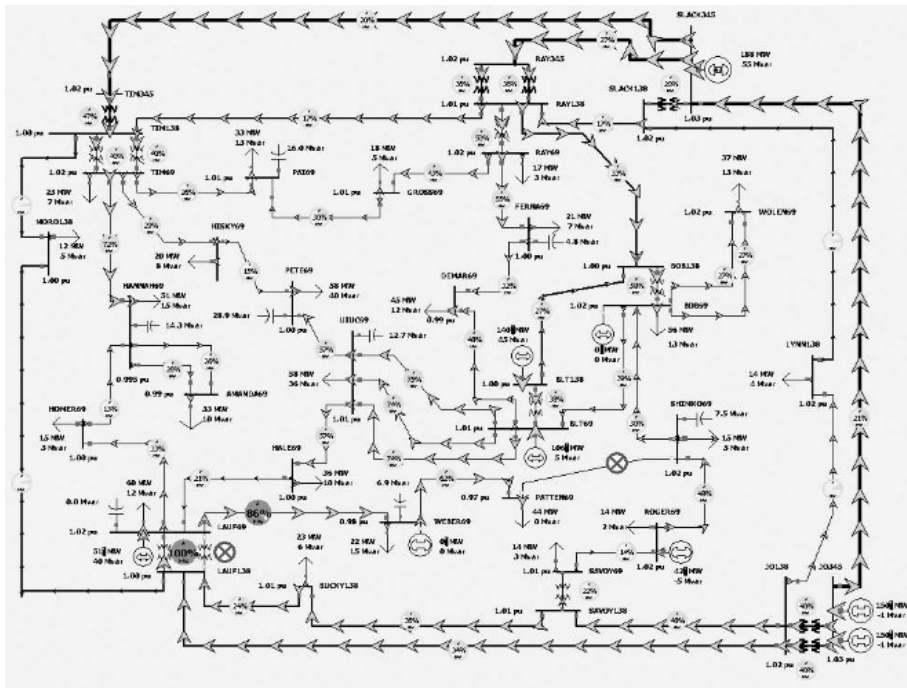


FIGURE 6.10 Screen for Example 6.15

PATTEN69 and SHIMKO69. This causes a flow of 116.2 MVA on the remaining 138/69 kV transformer at LAUF. Since this transformer has a limit of 101 MVA, it results in an overload at 115%. Redispatch the generators in order to remove this overload.

SOLUTION There are a number of solutions to this problem, and several solution techniques. One solution technique would be to use engineering intuition, along with a trial and error approach (see Figure 6.11). Since the overload is from the 138 kV level to the 69 kV level, and there is a generator directly connected to at the LAUF 69 kV bus, it stands to reason that increasing this generation would decrease the overload. Using this approach, we can remove the overload by increasing the Lauf generation until the transformer flow is reduced to 100%. This occurs when the generation is increased from 20 MW to 51 MW. Notice that as the generation is increased, the swing bus (SLACK345) generation automatically decreases in order to satisfy the requirement that total system load plus losses must be equal to total generation.



An alternative possible solution is seen by noting that since the overload is caused by power flowing from the 138 kV bus, decreasing the generation at JO345 might also decrease this flow. This is indeed the case, but now the trial and error approach requires a substantial amount of work, and ultimately doesn't solve the problem. Even when we decrease the total JO345 generation from 300 MW to 0 MW, the overload is still present, albeit with its percentage decreased to 105%.

An alternative solution approach would be to first determine the generators with the most sensitivity to this violation and then adjust these (see Figure 6.12). This can be done in PowerWorld Simulator by selecting **Tools, Sensitivities, Flows and Voltage Sensitivities**. Select the LAUF 138/69 kV transformer, click on the **Calculate Sensitivities** button, and select the Generator Sensitivities tab towards the bottom of the dialog. The “P Sensitivity” field tells how increasing the output of each generator by one MW would affect the MVA flow on this transformer. Note that the sensitivity for the Lauf generator is -0.494 , indicating that if we increase this generation by 1 MW the transformer MVA flow would decrease by 0.494 MVA. Hence, in order to decrease the flow by 15.2 MVA we would expect to increase the LAUF69 generator by 31 MW, exactly what we got by the trial and error approach. It is also clear that the JO345 generators, with a sensitivity of just 0.0335, would be relatively ineffective. In actual power system operation these sensitivities, known as generator shift factors, are used extensively. These sensitivities are also used in the Optimal Power Flow (introduced in Section 11.5). ■

FAST DECOUPLED POWER FLOW

Contingencies are a major concern in power system operations. For example, operating personnel need to know what power-flow changes will occur due to a particular generator outage or transmission-line outage. Contingency information, when obtained in real time, can be used to anticipate problems caused by such outages, and can be used to develop operating strategies to overcome the problems.

Fast power-flow algorithms have been developed to give power-flow solutions in seconds or less [8]. These algorithms are based on the following simplification of the Jacobian matrix. Neglecting $\mathbf{J}_2(i)$ and $\mathbf{J}_3(i)$, (6.6.6) reduces to two sets of decoupled equations:

$$\mathbf{J}_1(i)\Delta\delta(i) = \Delta\mathbf{P}(i) \quad (6.9.1)$$

$$\mathbf{J}_4(i)\Delta\mathbf{V}(i) = \Delta\mathbf{Q}(i) \quad (6.9.2)$$

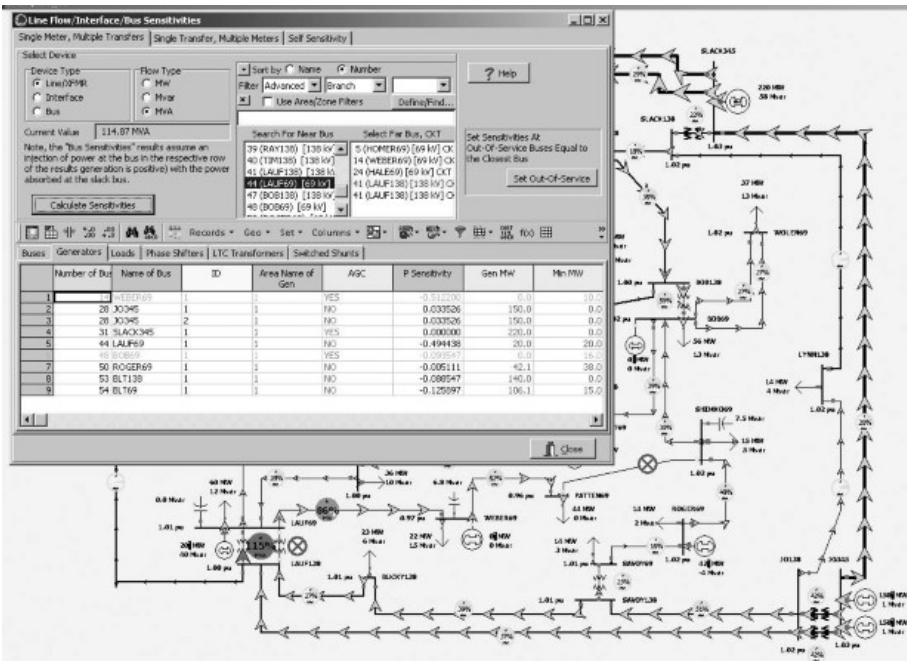


FIGURE 6.12 Example 6.15 Flow Sensitivities Dialog

The computer time required to solve (6.9.1) and (6.9.2) is significantly less than that required to solve (6.6.6). Further reduction in computer time can be obtained from additional simplification of the Jacobian matrix. For example, assume $V_k \approx V_n \approx 1.0$ per unit and $\delta_k \approx \delta_n$. Then J_1 and J_4 are constant matrices whose elements in Table 6.5 are the negative of the imaginary components of Y_{bus} . As such, J_1 and J_4 do not have to be recalculated during successive iterations.

The above simplifications can result in rapid power-flow solutions for most systems. While the fast decoupled power flow usually takes more iterations to converge, it is usually significantly faster than the Newton-Raphson algorithm since the Jacobian does not need to be recomputed each iteration. And since the mismatch equations themselves have not been modified, the solution obtained by the fast decoupled algorithm is the same as that found with the Newton-Raphson algorithm. However, in some situations in which only an approximate power-flow solution is needed the fast decoupled approach can be used with a fixed number of iterations (typically one) to give an extremely fast, albeit approximate solution.

THE “DC” POWER FLOW

The power-flow problem can be further simplified by extending the fast decoupled power flow to completely neglect the Q-V equation, assuming that the voltage magnitudes are constant at 1.0 per unit. With these simplifications the power flow on the line from bus j to bus k with reactive X_{jk} becomes

$$P_{jk} = \frac{\delta_j - \delta_k}{X_{jk}} \quad (6.10.1)$$

and the real power balance equations reduce to a completely linear problem

$$-B\delta = P \quad (6.10.2)$$

where B is the imaginary component of the of Y_{bus} calculated neglecting line resistance and excepting the slack bus row and column.

Because (6.10.2) is a linear equation with a form similar to that found in solving dc resistive circuits, this technique is referred to as the DC power flow. However, in contrast to the previous power-flow algorithms, the DC power flow only gives an approximate solution, with the degree of approximation system dependent. Nevertheless, with the advent of power

system restructuring the DC power flow has become a commonly used analysis technique.

EXAMPLE 6.17

Determine the dc power-flow solution for the five bus system from Example 6.9.

SOLUTION With bus 1 as the system slack, the B matrix and P vector for this system are

$$B = \begin{bmatrix} -30 & 0 & 10 & 20 \\ 0 & -100 & 100 & 0 \\ 10 & 100 & -150 & 40 \\ 20 & 0 & 40 & -110 \end{bmatrix} \quad P = \begin{bmatrix} -8.0 \\ 4.4 \\ 0 \\ 0 \end{bmatrix}$$

$$\delta = -B^{-1}P = \begin{bmatrix} -0.3263 \\ 0.0091 \\ -0.0349 \\ -0.0720 \end{bmatrix} \text{ radians} = \begin{bmatrix} -18.70 \\ 0.5214 \\ -2.000 \\ -4.125 \end{bmatrix} \text{ degrees}$$

To view this example in PowerWorld Simulator open case Example 6_17 which has this example solved using the DC power flow (see Figure 6.14). To view the DC power flow options select **Options, Simulator Options** to show the PowerWorld Simulator Options dialog. Then select the Power Flow Solution category, and the DC Options page.

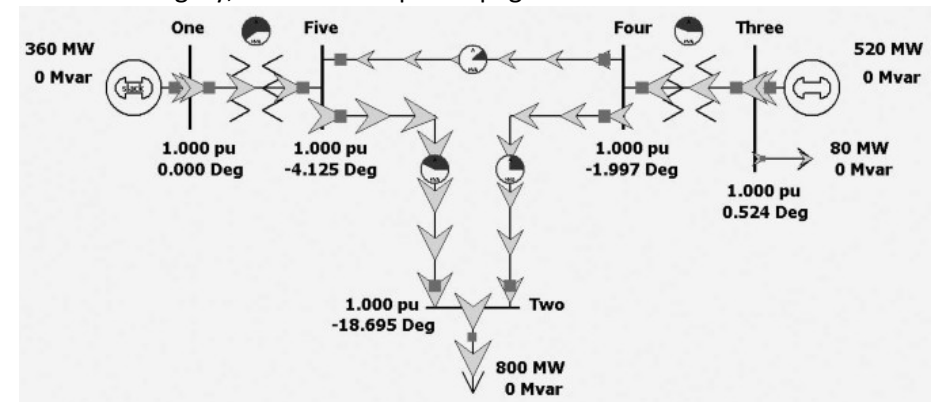


FIGURE 6.14 Screen for Example 6.17

